UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

1763540201

THINKING SKILLS **9694/32**

Paper 3 Problem Analysis and Solution **May/June 2011**

1 hour and 30 minutes

Additional Materials: Answer Booklet/Paper
Electronic Calculator

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES
Calculators should be used where appropriate.

Answer **all** the questions.
Start each question on a new answer sheet.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question.
The total of the marks for this paper is 50.

This document consists of **8** printed pages and **4** blank pages.

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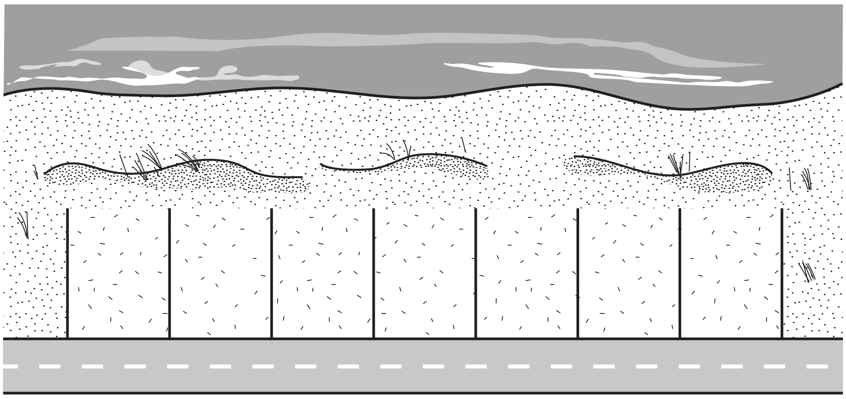
UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 *Study the information below and answer the questions. Show your working.*

A developer has obtained an isolated stretch of shore-front land, and intends to divide it up into seven building plots and build a luxury villa on each one. Designs will be selected and arranged to maximise his profit. The local planning regulations attempt to encourage variety, and do not permit a villa to have the same design as those next door or two doors away.



There are five possible designs available. In ascending order of profit when sold, these are *Lismore* (**L**), *Murwillumbah* (**M**), *Nimbin* (**N**), *Proserpine* (**P**), and the most expensive, *Rockhampton* (**R**).

(a) Give an example of an arrangement of villas that would maximise his profit. (List the letters representing the designs for the seven villas in a row.) [2]

Closer reading of the planning regulations reveals that a house which is the mirror image does **not** count as "the same". A 'reversed' design would generate just as much profit, and not require any extra work, just turning the plans over. We can distinguish the reversed version by adding the symbol *, e.g. having **M** and **M***. (Reversing again only brings us back to where we started.)

(b) **(i)** If reversed designs are permitted, how many of each type of villa will there be in the most profitable combination? (Remember that **L** and **L***, for example, are not considered to be the same type of villa.) [1]

(ii) Give **two** examples of how these might be arranged. [2]

The local council is also concerned that only building villas for the rich is not a good idea, and points out that there is a requirement for at least 20% of the housing in any development to be what they call 'affordable'. Only the *Lismore* style would qualify as affordable. In order to accommodate this requirement, the developer decides to divide his stretch of land into nine plots, and build enough *Lismore* villas to qualify.

(c) **(i)** How many *Lismore* villas (or mirror images) will he need to include? [1]

(ii) How many of each type of villa will there now be in the most profitable combination? [1]

(iii) Give **one** example of how these might be arranged. [1]

The council decide to change their planning regulations slightly, and now refuse to accept any reversed villas.

(d) Give **three** examples of arrangements for the nine plots that would maximise the developer's profit. [2]

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[Question 2 is printed on the next page]

2 *Study the information below and answer the questions. Show your working.*

The 'Universal Product Code' is used throughout the world to provide a consistent and unique identifying number for items bought in shops. The barcode on each product has thirty black bars printed on a white background so that items can be scanned by machine. If the scan does not work for some reason, human operators can input the digits printed underneath. Six digits are printed under the right half, six under the left, and an extra digit is printed to the lower left.

4 007817 504598

(a) How many gaps are there between the bars? [1]

Of the thirty bars, the two at each end and the two in the middle are always the same, but the rest are associated with the corresponding digits printed below. Something in the barcode must enable the machine to determine whether the barcode has been read from left to right or from right to left.

Each of the digits printed underneath the code is represented by two bars and two spaces, having a combined width of seven units. The width of each bar and each gap is always a whole number of units.

Different digits are distinct and cannot be confused with each other. For example, the digit **0** is **always** represented by the following pattern when it appears to the **right** of the middle two bars: a three unit wide **B**lack bar, a two unit wide **W**hite gap, a single unit wide **B**lack bar, and a single unit wide **W**hite gap. This can be written as:

[BBBWWBW]

However, if a digit **0** appears to the **left** of the middle two bars, it can be written in two different ways:

[WWWBBWB] or **[WBWWBBB]**

(b) Given this information about **0**, identify a pattern of two bars and two spaces, starting with **B**, that could **never** represent a digit on the right-hand side. Explain why it could not be found on the right-hand side. [3]

The digit **1** on the right-hand side would appear as:

[BBWWBBW]

(c) What are the two forms that the pattern for the digit **1** could take on the left-hand side? [2]

The reason there are two possible patterns for each of the digits on the **left**-hand side is to encode the extra digit (which is printed on the lower left). This extra digit is represented in the barcode by the combination of which of the two possible patterns is used for each of the six digits.

Suppose that one type of pattern is called *x* and the other type is called *y*. Then, when the extra digit is **4**, for example, the patterns for the six digits on the left-hand side are always used in the order *xyxxyy*. (This must not interfere with the ability of the machine to know which way round the barcode is read.)

Currently the extra digit can only be one of the numbers from 0 to 9. In the future there may be more combinations needed on the left-hand side.

(d) What is the total number of such combinations that have not yet been allocated, but would be safe to use because they would not lead to possible ambiguity in reading the barcode? [4]

3 *Study the information below and answer the questions. Show your working.*

Lumberjacks used to take the logs they had cut down to the sawmills by tying them together in rafts and floating them downriver.

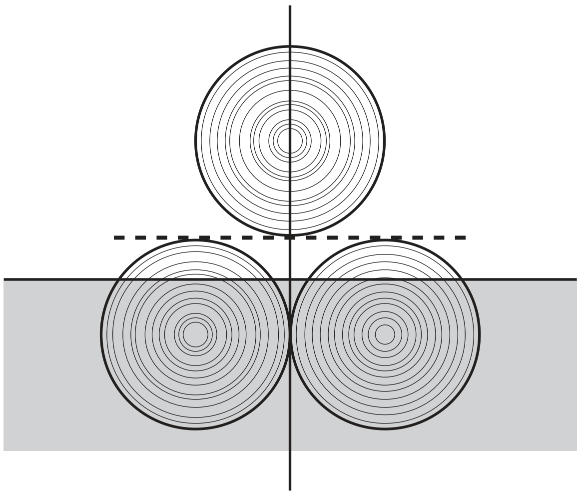
To simplify the model of this process you must assume that the logs are all identical cylinders lying parallel to each other. The flat timbers which separate the layers of logs (shown as a dotted line on the diagram) can be ignored for calculations.

The density of the logs is such that 59% of the volume of the raft is always underwater and 41% above it.

For stability, the lumberjacks constructed rafts according to the following constraints:

- Any layer of logs must have fewer logs than any layer below.
- The only place that a log on a higher layer can be put is directly above the point where two logs touch in the layer below.
- The cross-section of the raft has to have a vertical line of symmetry as shown in the diagram.
- There must be no gaps in the bottom layer.

The simplest two-layer raft is shown below. This raft will be referred to as a “2 + 1” – the numbers representing the number of logs in successive layers.



Line of symmetry

The proportion of the lower logs submerged can be calculated as shown:
Taking the volume of one log to be 1 unit, then the amount below the water is
59% of 3 units = (0.59) x 3 units = 1.77 units.

The lower logs have a combined volume of 2 units, so the percentage of them which is submerged is (1.77/2) x 100% = 88.5%.

(a) Calculate the percentage of the lower logs that will be submerged in a “3+2” raft. [2]

(b) Considering only two-layer rafts, what is the smallest total number of logs which could result in submerging the entire bottom row? Justify your answer. [3]

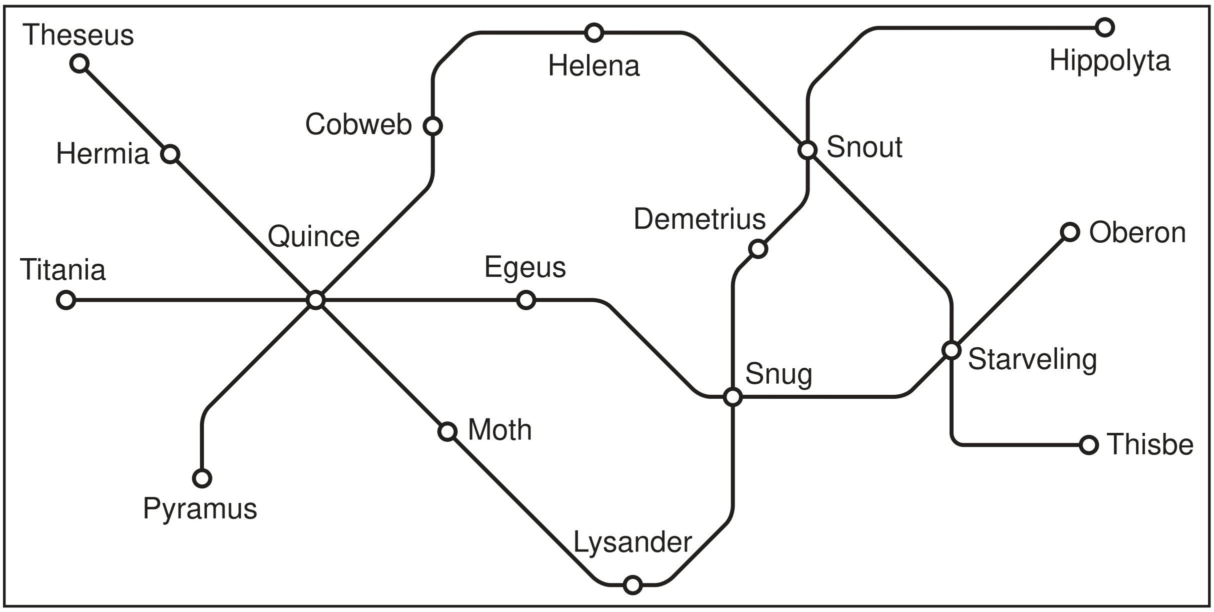
(c) If a certain journey requires rafts to be no more than 10 logs wide and two layers high, what is the largest number of dry logs (ones that are completely above the waterline) that can be transported? Justify your answer. [3]

(d) In how many ways can 8 logs be transported, if rafts can be of any height (but still must obey the four constraints given at the beginning)? State the arrangement of logs for each of the different ways it can be done. [3]

(e) What is the narrowest raft that can transport exactly 8 dry logs, if the rafts can be of any height? You **must** state the arrangement of logs in your answer. [4]

4 *Study the information below and answer the questions. Show your working.*

This is a map of the tram system in Emmendy, together with timetables for the three lines. The trams in Emmendy always run to timetable, and the time it takes between stations is consistent throughout the day.



Theseus – Hippolyta

Theseus	06:00	06:50		22:10	23:00	Hippolyta	06:00	06:50		22:10	23:00
Hermia	06:07	06:57		22:17	23:07	Snout	06:11	07:01		22:21	23:11
Quince	06:18	07:08		22:28	23:18	Demetrius	06:15	07:05		22:25	23:15
Moth	06:27	07:17	then every 20 mins until	22:37	23:27	Snug	06:21	07:11	then every 20 mins until	22:31	23:21
Lysander	06:38	07:28		22:48	23:38	Lysander	06:31	07:21		22:41	23:31
Snug	06:48	07:38		22:58	23:48	Moth	06:42	07:32		22:52	23:42
Demetrius	06:54	07:44		23:04	23:54	Quince	06:51	07:41		23:01	23:51
Snout	06:58	07:48		23:08	23:58	Hermia	07:02	07:52		23:12	00:02
Hippolyta	07:09	07:59		23:19	00:09	Theseus	07:09	07:59		23:19	00:09

Titania – Oberon

Titania	06:00	07:00		22:00	23:00	Oberon	06:00	07:00		22:00	23:00
Quince	06:09	07:09		22:09	23:09	Starveling	06:06	07:06		22:06	23:06
Egeus	06:17	07:17	then every 45 mins until	22:17	23:17	Snug	06:14	07:14	then every 45 mins until	22:14	23:14
Snug	06:24	07:24		22:24	23:24	Egeus	06:21	07:21		22:21	23:21
Starveling	06:32	07:32		22:32	23:32	Quince	06:29	07:29		22:29	23:29
Oberon	06:38	07:38		22:38	23:38	Titania	06:38	07:38		22:38	23:38

Pyramus – Thisbe

Pyramus	06:00	06:45	then every 30 mins until	22:15	23:00	Thisbe	06:00	06:45	then every 30 mins until	22:15	23:00
Quince	06:12	06:57		22:27	23:12	Starveling	06:07	06:52		22:22	23:07
Cobweb	06:22	07:07		22:37	23:22	Snout	06:18	07:03		22:33	23:18
Helena	06:29	07:14		22:44	23:29	Helena	06:27	07:12		22:42	23:27
Snout	06:38	07:23		22:53	23:38	Cobweb	06:34	07:19		22:49	23:34
Starveling	06:49	07:34		23:04	23:49	Quince	06:44	07:29		22:59	23:44
Thisbe	06:56	07:41		23:11	23:56	Pyramus	06:56	07:41		23:11	23:56

On all three lines fares are charged at the same rate between each pair of consecutive stations (known as stages) as follows:

Peak	**Off-peak**
75 cents per stage	40 cents per stage

Peak fares apply to the whole of a ride on a particular tram where the passenger gets on the tram between 07:30 and 09:30 inclusive and between 16:00 and 18:00 inclusive.

All three lines operate independently, so a separate ticket must be used whenever a passenger gets on a tram. For instance, a passenger travelling from Theseus to Hippolyta who chooses to change at Quince and Snug must buy three separate tickets (Theseus to Quince, Quince to Snug and Snug to Hippolyta: 2 stages, 2 stages and 3 stages respectively).

(a) How long does the journey from Quince to Starveling take

(i) on the Titania – Oberon line? [1]

(ii) on the Pyramus – Thisbe line? [1]

(b) Nicola and Frances arranged to meet at Snout station at 11:00 yesterday morning. Nicola arrived at 11:00 precisely, but had to wait 45 minutes before Frances turned up.

How many trams passed through Snout station while Nicola was waiting for Frances? [2]

(c) Peter has boarded the tram that will leave Thisbe at 06:45. His destination is Lysander, and he intends to change lines at Starveling and Snug.

(i) What time will he arrive at Lysander (assuming he will take the first available tram at both Starveling and Snug)? [3]

(ii) How much will his three tickets cost altogether? [1]

(d) It is 15:25 and Robin has just arrived at Cobweb station. He needs to get to Snug before 16:30.

(i) How many different routes are available to Robin? Give the arrival times at Snug for all the routes you consider. [5]

(ii) How much will it cost him if he takes the cheapest option? [2]

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